Thermohydraulic Instabilities in a Parallel-Tube System

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ABSTRACT

Thermohydraulic instabilities are very common phenomenon in fluid flow systems. They may occur in any flow regime, in any system, under natural or forced circulation, but their special group are two-phase instabilities. This type of instabilities occur in a two-phase flow regime or in a single-phase flow regime close to the saturation state. In every case, oscillations of flow rate and system pressure are undesirable [1]. Their possible consequences are: reactor control problems, an elevated risk of heat transfer surface burnout, thermal waves and mechanical vibrations.

In this paper, flow oscillations at the phase transition are addressed. A loop system consisting of parallel tubes is simulated in the SIMMER-V code. SIMMER-V is a thermohydraulic code originating from AFDM multi-phase CFD software. SIMMER-V is currently developed at the French Alternative Energies and Atomic Energy Commission (CEA) and the Japan Atomic Energy Agency (JAEA) in the frame of the joint France-Japan R&D collaboration. It is mainly used for SFR calculations with focus on severe accident loss-of-flow sequences.

For multiphase calculations, SIMMER-V uses the popular technique of regime mapping to represent the flow topologies on a void fraction/entrainment map [2]. There are few validation studies that include the specific situation of two-phase instability and behaviour at the two-phase flow regime boundaries. According to the driving mechanisms of different oscillation types, one can differentiate static and dynamic oscillations, when the later constitute the major group of oscillations is real systems. In such systems, different types of oscillations are favoured at certain range of the boundary conditions: heat flux, mass flow rate, inlet subcooling and local flow restrictions.

Keywords: *instabilities, SIMMER-V, multi-channel system, in-phase, out-of-phase*

1 INTRODUCTION

Investigation of flow instabilities is addressed to all the types of fluid-flow systems, from very simple to very complex ones: heat exchangers, thermosiphons, evaporators and finally nuclear reactors. Several instability events were observed in already constructed and operating nuclear power plants in 1980s. Probably the most famous event happened in LaSalle 2 General Electric station in the USA in 1988. A human error during the standard maintenance caused a shutdown of both recirculation pumps and drop of the flow from 76% to 29% of the nominal value [3]. Moderately high core power and low-flow conditions caused strong power oscillations and led to a reactor scram when the power level achieved 118% of the full power, in about 7 minutes after the pump trip. However, it was not the first instability event in a boiling-water reactor (BWR). In 1982 in another General Electric power plant Caorso, Italy, during the startup of the reactor similar operating conditions were achieved: moderate power (53,5%) at low flow rate (38% of the nominal value). Unstable conditions caused a scram signal at 120% of the full power [4]. More events occurred in Swedish power plants: Forsmark 1, Oskarshamn 2 and 3, Ringhals 1, Finnish: TVO-I, German: Siemens station Isar 1, Mexican: Laguna Verde 1 and American: WNP 2 and Perry [4].

1.1 Relevance of Instabilities to the ASTRID Project

This study of the thermohydraulic oscillation is conducted in framework of the CEA research project on ASTRID design. Even though the project was stopped from the industrial perspective in 2019, the ongoing research focuses on codes development in validation for a sodium-cooled reactor construction in the second half of the 21st century.

One aspect of the ASTRID safety assessment is a scenario of an unprotected loss-of-flow (ULOF) accident. In ULOF sequence, one can distinguish a short- and long-term phases, in which void effect, re-compaction, re-criticality or core melt, confinement rupture and decay heat removal play the crucial role, respectively.

The flow instabilities address the short-term of the ULOF scenario [5]. After a pump coastdown, natural circulation should be enough to bring the reactor to a safe cool-down. A postulated ASTRID safety feature should ensure that the core overall sodium void effect is near zero and should be even negative. It is a special feature of a CFV (French: low-void worth) core, in which a fuel section is divided in the middle by a horizontal fertile plate [6]. Additionally, outer fuel zone is larger than the inner zone, which enhances the neutron leakage. These modifications are postulated to avoid the massive voiding of the core. At the same time, the two-phase flow upon the core should be maintained in order to enhance the neutron leak. The safety demonstration assumes in this zone a stable boiling can be maintained at least temporarily, with a stable boiling boundary, in order to provide a grace period for accident consequences mitigation.

2 THERMOHYDRAULIC INSTABILITIES

Thermohydraulic instabilities can be classified in several ways, however most often they are categorized into the static and dynamic oscillations classes. Among them, they differ by the mechanism and observable properties, like oscillation period and pressure drop variations. The phenomenological approach allows studying them as such, in simple systems in which it is possible to distinguish each mechanism. Systems which are more complex usually combine several oscillation types.

2.1 Phenomenological Approach

Generally, average channel flow rate is in increasing linear dependence with heat flux [7] with a slope increasing for lower subcooling. This condition should ensure steady flow in all points of the system. However, it is often not the case due to multiple feedbacks related to the pressure drop. Such a situation leads to flow oscillations and instabilities.

According to the mechanisms driving the oscillations, one can differentiate static, when a disturbance introduced to the system brings it to new operating conditions from which the system tends to a stable operating point (new equilibrium), and dynamic oscillations, when an introduced perturbation of flow or power results in a feedback (or multiple feedbacks) leading to repetitive variations of parameters.

Among static oscillation, there is a Ledinegg type instability [8], flow excursion (hysteresis from Ledinegg instability) [9] and flow pattern transition [10]. Dynamic oscillations occur when due to the feedbacks, a disturbance is not damped but reflected. In global view, three main types of dynamic instabilities can be distinguished [11]: density-wave oscillation (DWO), pressure-drop oscillation (PDO) and thermal oscillation. Some authors include also other types: acoustic oscillations which are born in fact as resonant pressure waves [1] and flashing. Another type is geysering [12], classified as a quasi-static oscillation. Fluid transport in this mechanism depends on the non-heated chimney above liquid reservoir. Brennen [13] classified this type of oscillation a chugging and condensation instability, and pointed another initiation event: forced steam inflow to the reservoir, which is different to fully natural convection in geological geysering phenomenon.

Flashing may be confused with geysering phenomenon, but it occurs in the systems with an unheated sections [14]. Flashing was described by Bragt and Hagen [15] who noticed that voiding in the riser (chimney) section occurs even in absence of boiling in the heated section, due to a decrease in hydrostatic pressure, which caused very rapid evaporation (flashing).

In the early years of instabilities research, flashing instability was confused with a density wave oscillation. This confusion with arises from the oscillation period which is equal to the time required for the single-phase flow to travel through the chimney [16]. Phase shift between the temperatures in flashing are also similar to DWO [17]. According to Fort and Laussier [18] density-wave models are able to predict most of the oscillatory phenomena in a BWR. Generally, DWOs are caused by multiple feedbacks between flow rate, vapour generation rate and pressure drop [7]. DWO covers a wide range of phenomena, according to Subki et al. [7], DWO can be low- and high-frequency oscillation, in both cases classified as a self-sustained instability induced by flashing. DWO can be detected by its peculiarity which is the negative friction coefficient only at a specific frequency. Thus, the instant flow increase is not an average flow increase but a flow increase at the given frequency [19]. A very similar oscillation exists at very high pressure. By Xiao et al. [20] it is called a 2nd DWO. Ruspini et al. [21] suggest to divide DWOs into categories dependent on their driving force, rather than operational point. This point of view has more physical and logical meaning but is less practical in experimental considerations. He distinguishes DWO: i) due to gravity, ii) due to friction, iii) due to momentum.

A pressure-drop oscillation has been first studied and named by Callahan et al. [22] and found as an interaction of a flow excursion and the tank compressibility. Pressure drop oscillation can develop when the slope of an internal characteristics is negative [1], [23] and external characteristics is steeper than the internal. A prerequisite for this type of oscillation, according to [1], is a presence of an external or internal compressible volume, which in real systems is usually fulfilled by a surge tank. A particular feature of this oscillation type is low frequency compared to DWO. It is determined by the volume and compressibility of the vapor in the system.

Thermal oscillations are fluctuations originating in PDO [1], with a name relating to the wall temperature fluctuations and occur where solid interacts with a fluid. They have been experimentally confirmed in systems of relatively low total flow rate, in which fluid at the tube exit was superheated or of very high quality [24]. A phase change is followed by variation of heat transfer coefficient variation between the heater and fluid. Due to that, heat input changes is time, while heat generated by the heater is constant and imposed by external control. When the wall contacts with a liquid, heat transfer coefficient is relatively high and wall temperature decreases.

2.2 Factors Favouring Instabilities

Studies of systems under natural and forced circulation showed significant differences in their stability. A pump presence in the system results in a positive damping of oscillations. In a single-channel system Hayama [25] confirmed that the system is more stable under forced than natural circulation. Hydrostatic flow oscillations come from variations of a driving force and rapid void generation [24]. Void fraction in the non-heated channel section [2] changes periodically in time leading to hydrostatic flow changes, where the flow is not forced by any external factor.

Subki et al. [7] noticed natural circulation oscillation only at atmospheric pressure, while other authors confirmed that pressurizing the system up to 2 bar [26] or 5 bar [27], [28] avoids hydrostatic natural circulation oscillations in a reactor systems.

Factors which stabilize the system are uniform axial distribution of power and local friction coefficient at the channel inlet. If the cross-flows are possible, a multichannel system can be then more stable than a single channel [1]. Complex system used for experiments are sometimes equipped with a by-pass, which allows the flow to omit the heated section flowing by an isolated tube and join the heated flow above a riser. It was found that in such systems, its usage should be limited as it destabilizes the flow in parallel channels [1].

Very often, not only physical mechanism is involved in evoking an oscillation. The Ledinegg oscillation is commonly combined with DWO, and a DWO-PDO interaction is possible. Oscillations may form different shapes: intermittent oscillation [11], [33], [34], sinusoidal oscillation [16], [29], [30], compound oscillation [30], double-peak oscillation [29], irregular but periodical oscillation [29].

2.3 Multi-channel and Large Systems

In multichannel systems some oscillations like DWO is more likely to happen than in a single channel [20] and under some conditions can start earlier than PDO.

Out-of-phase thermal oscillations have been observed in a two-tube systems by Akagawa et al. [24] but until now, this phenomenon has not been precisely studied for multi-channel systems. In a system with equally heated channels, Xiao et al. [20] observed oscillation only out-of-phase, with a constant total mass flow rate.

A three-channel system in the experiment of Akagawa et al. [24] oscillated in a way two channels being in-phase and the third out of-phase with them. Darcy [1] observed the same type of oscillations and two more: he observed three channels oscillating in-phase, as well as two channels oscillating out-of-phase and the third channel was steady.

Stability of a large system, like a reactor core, depends on fraction of power transferred to the fluid in one- and two-phase regions. Oscillations in large systems can be divided into two categories which can coexist [18]: in-phase (global) oscillations and out-of-phase (regional, channel-to-channel [20]) oscillations. Corewide instability (at least in a boiling water reactor (BWR), for which it has been investigated) is more likely to appear when:

- void fraction is high;
- cosine power shape in many reactors is deformed: when it is bottom-peaked, it leads average void fraction increase;
- the most unstable channel dominates the reactor behavior – thus multi-channel systems are favourably symmetrical;
- low void velocity – it increases the delay time;
- decrease of inlet subcooling causes increase of operating power (which has a destabilizing effect) and rises the physical boiling boundary in a core (density wave time delay decreases, system gains stability) – usually destabilizing effect is dominating;
- increased fuel gap conductance destabilizes the system by more heat deposition to coolant during a neutron flux oscillation of a particular frequency [19].

Out-of-phase mode of oscillation in water-cooled systems is more likely to happen when:

- geometric buckling is low; it concerns large cores [31];
- pressure drop across the channel is high – it increases the flow feedback;
- high flow rate;
- high friction in recirculation-loop;
- axial power shape significantly deflected towards the bottom – if peaking factor >1,6, out-of-phase mode is more probable; pressure drop is increased what increases the thermohydraulic gain;
- low single-phase friction – loose inlet orifices may result in DWO [19].

3 METHOD OF THE MULTI-CHANNEL SYSTEM ANALYSIS

3.1 SIMMER-V Thermohydraulic Code

The SIMMER code is multi-velocity-field, multi-phase, multi-component, Eulerian fluid-dynamics code. Its advanced three-dimensional (3D) version SIMMER-V is currently developed in the framework of France-Japan collaboration. SIMMER-V is under the ownership of Japan Atomic

Energy Agency (JAEA) and is co-developed at the French Alternative Energies and Atomic Energy Commission (CEA). The code originates from the American AFDM CFD code and is coupled with a structure model and a space-, time- and energy-dependent transport theory neutron dynamics code.

SIMMER-V uses a map technique to for modelling a two-phase flow. The by heat transfer coefficient and interfacial area and mass exchange correlations depend on a flow regime. The flow regime is determined for conditions on a two-dimensional map, for a channel (used for this study) or pool flow pattern, with the entrainment factor and void fraction as decisive parameters.

3.2 Analytical Stability Boundary Estimation

Several analytical studies of multi-channel systems have been presented in the past. Fukuda et al. used a matrix technique [32] and a perturbation technique was used by Guido et al. [33] for study of a system of two different channels. Coupled systems, like BWRs, were studied analytically by Peng et al. [34]. Guido et al. [35] developed a simple analytical model for a single and multiple channels. In this model, ordinary differential equations are written for two sections in each channel: for mass and energy conservation in a single- and two-phase length. It is a lumped parameter model where these regions have variable length. The model assumes homogeneous equilibrium model (HEM) for the two-phase flow and constant system pressure. The fluid enters the channel with a constant temperature and is heated with a constant and uniform heat flux. The subcooled boiling is not considered and friction losses are concentrated at the channels inlets and outlets.

The model leads to the set of equations which are dimensionless with two crucial dimensionless numbers: the subcooling number:

$$N_{SUB} = (h_F - h_i)\frac{v_{FG}}{v_F h_{FG}} \tag{1}$$

and the phase-change number:

$$N_{PCH} = \frac{Q}{W} \cdot \frac{v_{FG}}{v_F h_{FG}} \tag{2}$$

where: Q – heating power, W – mass flow rate, v – specific density, h - specific enthalpy, subscripts: F – saturated liquid, FG – transfer from liquid to vapor

The final conclusion on the boundary of a DWO is:

$$N_{PCH} - N_{SUB} < \frac{\tau}{2}\left(1 + \frac{1}{N_{SUB}}\right) - \frac{5}{2} + \left\{\left[\frac{\tau}{2}\left(1 + \frac{2}{N_{SUB}}\right) - \frac{5}{2}\right]^2 + \tau\right\}^{0,5} \tag{3}$$

where: τ – friction parameter, for a single channel in a form:

$$\tau = \frac{2(K_i + K_e)}{K_e + 1} \tag{4}$$

and for multiple channels connected by common lower and upper plenum (Figure 1), for in- and out-of-phase oscillation modes, respectively:

$$\tau_T = \frac{2(K_I' + K_E' + K_i + K_e)}{K_E' + K_e + 1} \tag{5}$$

$$\tau_P = \frac{2(K_i + K_e)}{K_e + 1} \tag{6}$$

where: $K_I' = K_I\left(\frac{\sum_j A_j}{A_T}\right)^2$ and $K_E' = K_E\left(\frac{\sum_j A_j}{A_T}\right)^2$

It means that the definition of the instability inception for an out-of-phase mode is coherent with a single-channel definition. An additional boundary appears for the in-phase mode. The thresholds could be interchangeable depending on flow restrictions at the inlet and outlet from the common plena. If $\tau_T < \tau_P$, the system is more unstable for in-phase oscillations.

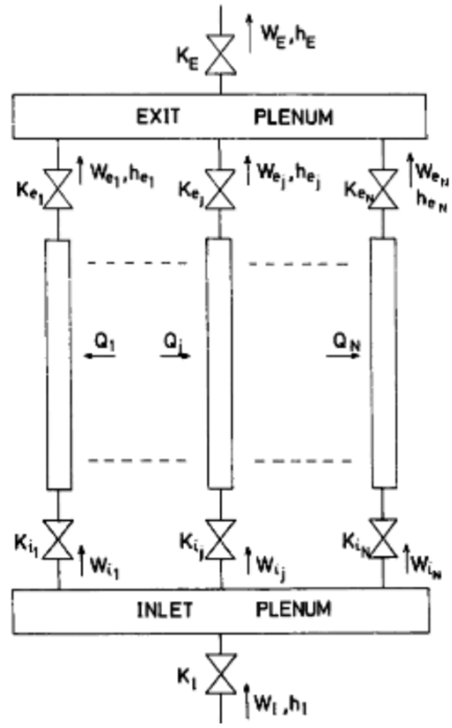


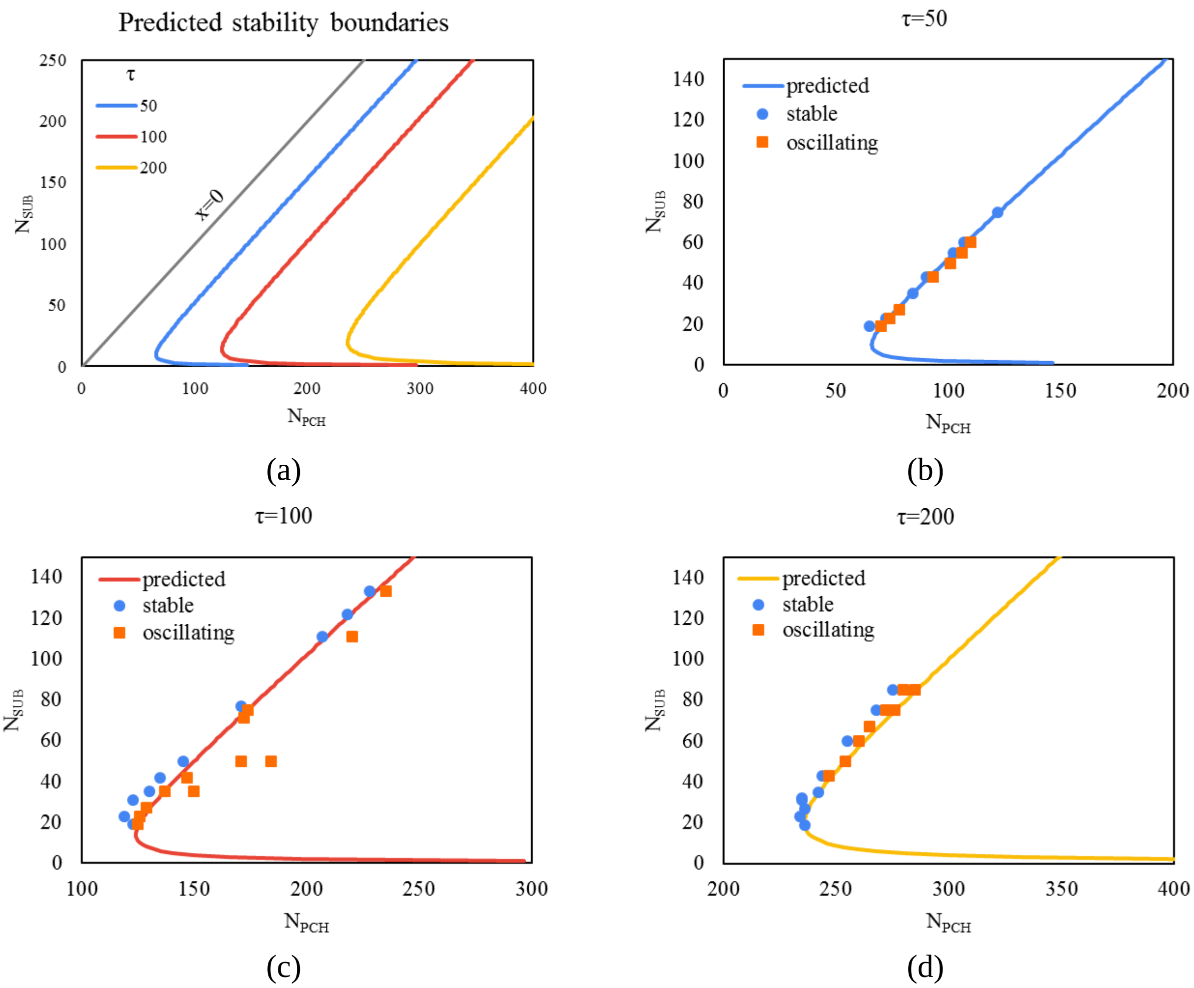
Figure 1: Scheme of a Multi-channel Systems with Variables [35]

4 RESULTS AND DISCUSSION

The system as in Figure 1 was represented in SIMMER-V. Thanks to the *virtual wall* elements, it was possible to close as many channels as needed for each simulation. At the moment, the standard SIMMER-V version with obligatory Cartesian meshing was used, which imposed meshing of the lower and upper plena in the vertical direction. Thus, we diverged from the assumption of a lumped model and obtained a solution with a radial velocity and temperature profile which had to be averaged.

Firstly, three values of the friction coefficient were selected to provide qualitative comparison of the stability boundary: $\tau = 50, \tau = 100$ and $\tau = 200$. Shapes of the L-curve showed that at higher τ, i.e. at higher channel inlet restriction, the two-phase stable zone is wider (Figure 2a). Each of these three systems was simulated at several level of subcooling. The points around the expected instability threshold were chosen first to find the location of the DWO onset.

The analytical model was observed to be in the best agreement at lower friction coefficient ($\tau = 50$), for which the instability threshold error is nearly zero (Figure 2b). At moderate τ (Figure 2c) the model brings a small error in the vicinity of the L-curve saddle point, predicting the system more stable than found by SIMMER-V. At higher friction coefficient an overestimation of the phase-change number for the oscillation onset is observed at higher subcooling (Figure 2d). The difference between analytical and SIMMER-V result systematically increases with the subcooling number increase. An explanation of this behavior is not obvious but it can be supposed that in this case the results are often affected by the two-phase friction model. According to Papini [36] a higher two-phase friction makes the channel more unstable, for example for Lockhart-Martinelli and Jones models.



(a) (b)

(c) (d)

Figure 2: Stability boundaries for a single channel: (a) predicted by Guido et al. [35], (b), (c) and (d) calculated in SIMMER-V and compared to the prediction

In a system with two channels, the behavior was more complex. For the chosen values $\tau_T = 70$ and $\tau_P = 120$ the system exhibited an intermediate behavior between in- and out-of-phase oscillations. Analytically obtained stability boundaries correspond to the SIMMER-V solution especially at low subcooling (Figure 3). At higher subcooling, small diversions from the in-phase threshold were observed, and no out-of-phase instability was obtained by SIMMER-V.

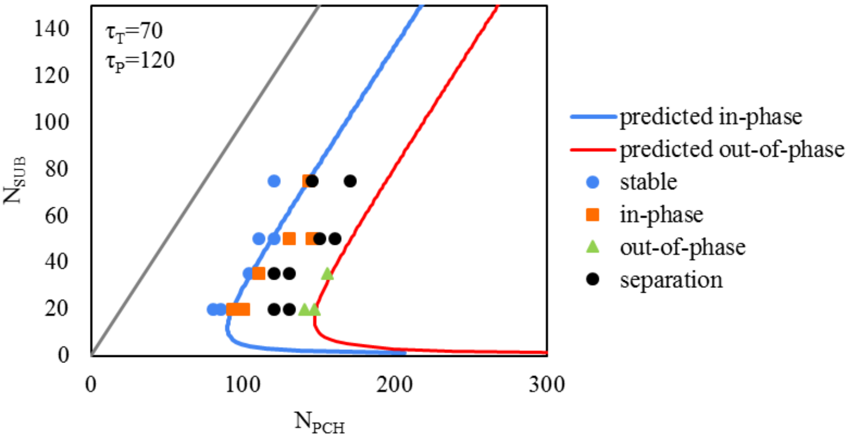
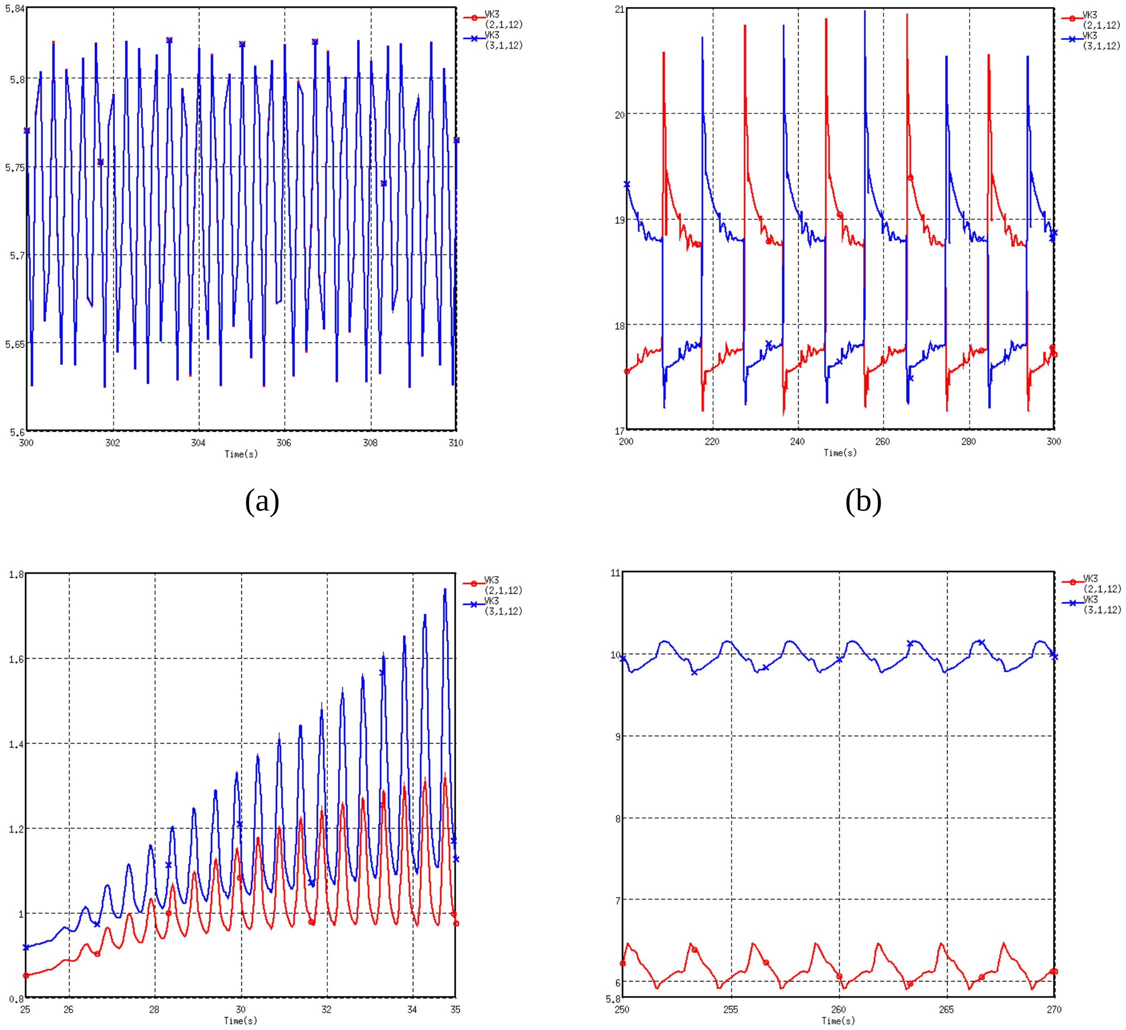


Figure 3: Predicted in- and out-of-phase DWO threshold boundaries with SIMMER-V results for a two-channel system

In a two-channel systems more diverse flow patterns were observed. At lower phase-change numbers the velocity and mass flow rates profiles oscillated with high frequency but relatively low amplitude (a few per cent of the average value) (Figure 4a). The out-of-phase oscillation occurred at higher heating powers, thus at higher flow velocities (at the same mass flow rate) and with larger amplitudes. Between these phenomena, intermediate stage of non-uniform flow distribution occurs. It is a known phenomenon [24], [37], [38] that in a parallel-tube system the flow does not have to be distributed evenly between the channels. After the full power was achieved, the flow rate of one channel goes up (Figure 4c). At high subcooling, out-of-phase oscillations are observed in such non-uniformly distributed flow (Figure 4d) which does not come back to the same average flow rate, as in Figure 4b. Some out-of-phase oscillations are not perfectly shifted by the 180°. Such a result is acceptable according to Rizwan-Uddin [39] who proved that the single-phase and two-phase pressure drop shift is not perfect.



(a) (b)

(c) (d)

Figure 4: Characteristic shapes of observed oscillations in a two-channel system: (a) high-frequency in-phase oscillation, (b) low-frequency out-of-phase oscillation, (c) moderate-frequency out-of-phase oscillation with a non-uniform flow distribution, (d) beginning of non-uniform flow distribution with high-frequency in-phase oscillations

5 CONCLUSION

The Guido et al. model corresponds perfectly to the behaviour of a single channel, especially at low inlet subcooling. At higher subcooling numbers SIMMER-V diverges from the analytical prediction, probably due to the flow regime map. Future work could include introducing a Friedel correlation for the two-phase multiplier [36] to better represent the two-phase friction.

The similar observation comes from the two-channel modelling, in which an in-phase boundary was found the same by analytical and numerical calculations, however out-of-phase oscillation prediction failed at higher subcooling. The analytical model does not include flow splitting issues and it is assumed *a priori* that flow rates in both channels are identical.

Potentially, similar modelling in SIMMER-V with the boundary coupling extension could be used to remove the simulation uncertainty from the plena meshing. Such a geometrical model would correspond better to the geometry assumed for the lumped parameter model derivation.

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SIMMER-V is developed in the framework of the Japan-France joint collaboration and continues to occupy an important position in the SFR R&D collaboration from 2020 to 2024.

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