Modelling of Diffusion Neutrons Flux

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ABSTRACT

The well-known law of attenuation of a narrow particle flow cannot be applied to a diffusion flow. Usually, differences are tried to be taken into account by applying the empirical build-up coefficient, but there is no theoretical justification for its value. An appropriate diffusion flow model is required to bridge this gap.

The process of diffusion flux formation in the medium where neutrons diffuse and near the boundary with the medium where they are absorbed is considered.

The distribution of neutron concentration nearby the boundary between the diffusion and absorption medium by iterative approximation is presented. The distribution depends on the neutron-physical properties of the diffusion medium. The obtained results are the foundation for further analytical study of the law of attenuation of diffusion flux.

Keywords: *neutron diffusion flux, attenuation, build-up coefficient, Monte Carlo model, neutron concentration distribution*

1 INTRODUCTION

The law of attenuation is the basis for calculations of radiation shields. However, it is important to remember the limitations of its use, because it correctly describes the weakening of a narrow (collimated) beam of radiation directed perpendicular to the shield. In practice, there is often a diffusion flux of particles, and this difference is usually tried to take into account by applying the empirical build-up factor (BUF). Theoretical substantiation of the BUF value is absent and is based on empirical processing of heterogeneous experimental data [1-4].

The study [5] compares linear and other formulas for calculating build-up factors that take into account only the thickness and properties of the shield, which depend on the radiation energy. There is a significant difference in estimates, which sometimes differ at times.

Significant remarks have been made in studies of BUFs for multilayer shielding [6, 7]. It is that BUFs is depended on the shielding layer order. However, for now, the importance of the sequence of shields materials is not reflected in the methods of calculating BUF. Based on this, we can conclude that the amount of attenuation in the shield depends not only on the shield properties, but also on what happens in front of the shield, which is the diffusion flux formed in front of it. Further research is devoted to the study of this problem.

2 FORMATION OF NEUTRON DIFFUSION FLUX IN A HOMOGENEOUS MEDIUM

To explore what is happening in front of the shield, consider a simple physical model. The shield in the form of an infinite plate made of a material that only absorbs neutrons is located after a layer of material that only scatters neutrons. This conditional separation of the ability to absorb and scatter neutrons is made for simplification. The flat source is located at a sufficient distance from the absorber plate, which allows forming a diffusion flux of neutrons. First, we assume that the neutron concentration in the whole diffusion medium is the same. An appropriate clarification will be made later.

In the process of diffusion, the neutron accidentally changes the direction of its motion. Between the scatter on the atoms of the medium, the neutron moves in a straight line, so its track is a broken line. The mean free path (mfp) length λ of a neutron depends on the neutron-physical properties of the medium. The probability that the next scattering in the medium where the diffusion occurs will occur at a certain distance x can be calculated by the formula

$$p_1(x) = \exp(-\Sigma x) = \exp(-x/\lambda) \qquad (1)$$

where

Σ, m^{-1} is the macroscopic cross-section of the nuclear scattering reaction for the medium material

$\lambda = 1/\Sigma, \mathrm{m}$ is the mean free path.

For a distance multiple of λ, the corresponding probability values are presented in Table. 1.

Table 1: Probability of free path length before the next scattering

x	λ	2λ	3λ	4λ	5λ	6λ
$p_1(x)$	0,3679	0,1353	0,0498	0,0183	0,0067	0,0025

Only about 5% of neutrons cover a distance of more than 3λ until the next scattering. Thus, 95% of subsequent scattering occurs in the volume of a sphere of radius R = 3λ around the point of previous scattering.

Similarly, it is possible to estimate the probability of subsequent scattering at a certain point after the neutron has passed the distance *R*. The probability of reaching the scattering point from the distance R is also determined by equation (1), but the number of possible previous scattering at such a distance increases in proportion to the surface area of the sphere of radius *R* around the scattering point. Thus, the distribution of neutrons that have covered the distance *R* from the place of previous scattering is determined by the equation (1) and presented on Figure 1.

$$p_2(x) = 4\pi R^2 \exp(-R/\lambda) = 4\pi\lambda^2 x^2 \exp(-x) \qquad (2)$$

where $x = R/\lambda$ distance in mfp

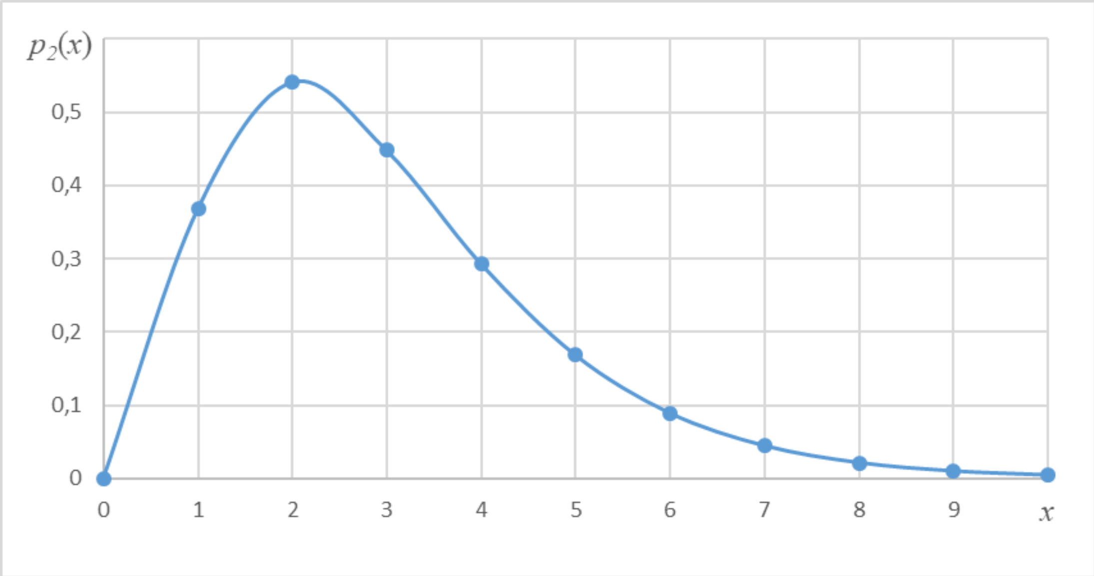

Figure 1: Distribution of neutrons reaching the scattering point from a distance x where the previous scattering took place

The maximum of the distribution can be defined as the extremum point of the function. The derivative of equation (2) as the product of two functions is equal to

$$p_2'(x) = 4\pi\lambda^2\,[2x\exp(-x) - x^2\exp(-x)] = 4\pi\lambda^2\exp(-x)(2x - x^2) \quad (3)$$

and becomes equal to zero at $x = 2$, which determines the maximum of the function $p_2(x)$.

The greatest contribution to the diffusion flux in a certain point is given by neutrons that had pre-scattering at a distance of about $R = 2\lambda$. The maximum of the distribution is because the number of such neutrons is already large enough, and the probability of overcoming the distance R is still quite large.

The cumulative number of neutrons entering the scattering point is proportional to the integral of $p_2(x)$. The value of the integral normalized to one for a homogeneous medium depending on x presented in Figure 2. Typical integrals used for calculation:

$$\int x^n \exp(ax)\,dx = \frac{1}{a}x^n\exp(ax) - \frac{n}{a}\int x^{n-1}\exp(ax)\,dx \quad (4)$$

$$\int x\exp(ax)\,dx = \frac{1}{a^2}\exp(ax)\,(ax-1) \quad (5)$$

Using (4) and (5) we obtain

$$I(x) = \int p_2(x)\,dx = \int x^2\exp(-x)\,dx = -x^2\exp(-x) + 2\int x\exp(-x)\,dx \quad (6)$$

$$\int x\exp(-x)\,dx = -\exp(-x)\,(x+1) \quad (7)$$

$$\int p_2(x)\,dx = -x^2\exp(-x) - 2\exp(-x)\,(x+1) = -(x^2+2x+2)\exp(-x) \quad (8)$$

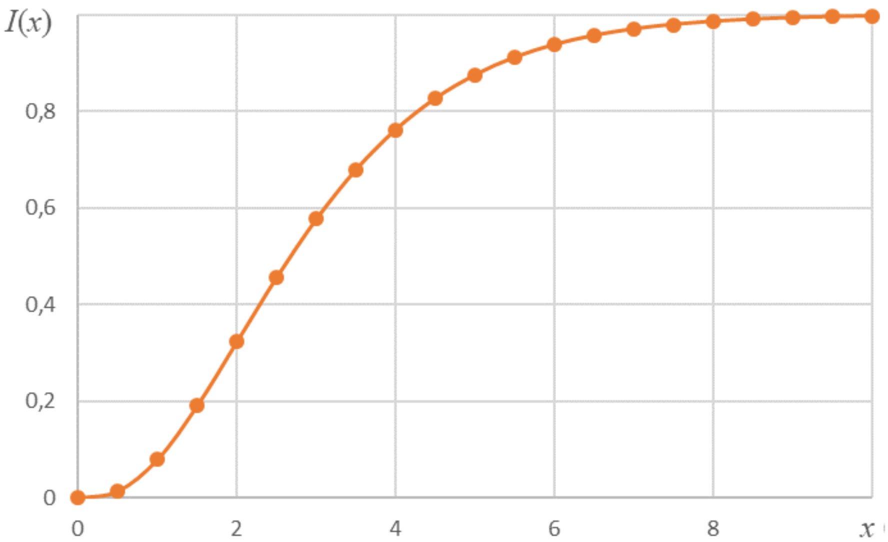

Figure 2: The cumulative number of neutrons that come the scattering point from distance $x = R/\lambda$ after the previous scattering (normalized to one).

Thus, the neutron diffusion flux at a certain point is determined by how much previous scattering occurs around it. Most of neutrons (95%) contributing to the diffusion flux density had pre-scattering within a sphere of radius R = 6,30 λ.

3 DISTRIBUTION OF NEUTRON DIFFUSION FLUX IN FRONT OF THE ABSORBING SHIELD

The distributions presented in Figure 1 and Figure 2 may be affected by the inhomogeneity of the environment in which neutron diffusion occurs. Recall that the material of the diffusion medium only scatters neutrons, and the shield material only absorbs them. The density of the neutron diffusion flux at a point near the boundary determined by the number of scatterings within a sphere of a certain radius around it.

The boundary between the media cuts off the sector of the sphere of previous scattering (see Figure 3), which is in the absorbing medium and in which there is no previous scattering. For points that are close to the boundary, the neutron diffusion flux density will be halved because the boundary bisects the sphere. This estimate based on the assumption that scattering is evenly distributed throughout the medium is therefore only a first approximation. The effect of reducing the neutron flux and the corresponding reduction in the number of scattering near the boundary amplifies itself.

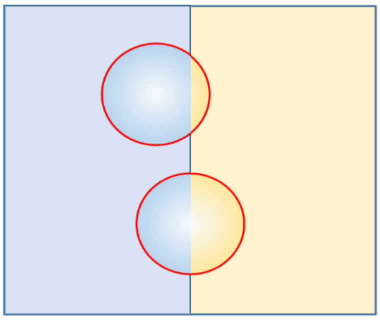

Figure 3: Scattering near the boundary with the neutron absorbing shield.

For a point located at a distance H from the boundary, the surface area of the sphere of radius R at which the previous scattering occurs shown on Figure 4. Boundary lines correspond to a complete sphere in a homogeneous medium and a hemisphere for a point on the boundary between two media. Dotted lines on Figure 4 correspond to points at a distance H. The limiting variants are the hemisphere at $H = 0$ and the sphere at $H = \infty$. Intermediate variants are calculated as the sphere surface except for the surface of the sphere sector, which is cut off by the boundary between the media, according to the formulas

$$S(x) = 4\pi x^2 \qquad x < H \tag{9}$$

$$S(x) = 4\pi x^2 - 2\pi x(x - H) \qquad x > H \tag{10}$$

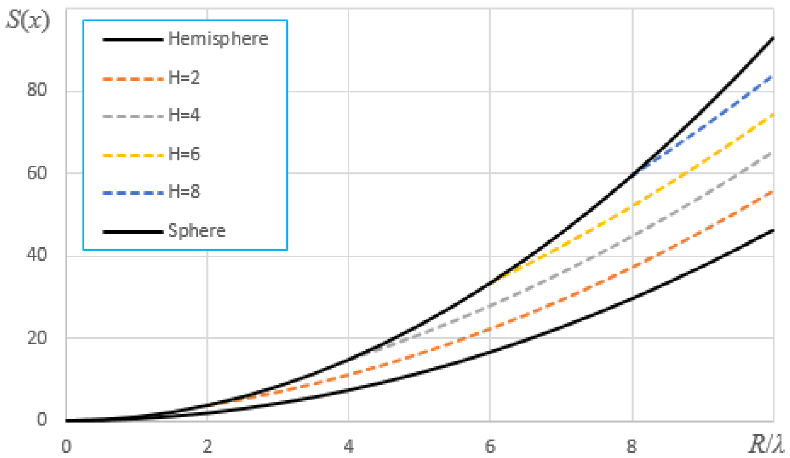

Figure 4: The surface area of a sphere of radius R on which preliminary scattering takes place for different distances from the boundary between the media H.

The number of neutrons coming to scattering point from the distance R / λ (see Figure 5) can be obtained taking into account the probability to pass the distance after the previous scattering. As expected, when approaching the boundary between media, there is a significant change in profile. At $H < 2$ there is not only a change in the shape of the profile but also a shift of the most probable value, which at the limit H is equal to $R / \lambda = 2$.

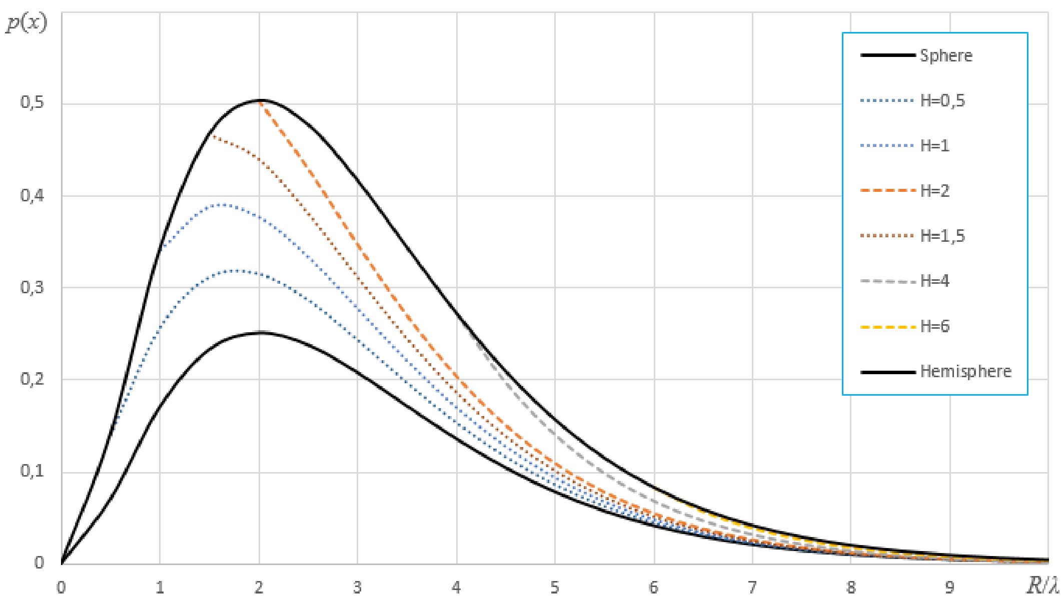

Figure 5: Distribution of several neutrons entering the point of view from the distance R/λ for different distances to the boundaries H.

An estimate of how the neutron concentration changes as it approaches the boundaries between the media can be obtained as the mean integral value of the distributions of the number of neutrons coming to the scattering point.

$$n(x) = \int_0^H 4\pi x^2 \exp(-x)\,dx - \int_H^\infty 2\pi x(x-H)\exp(-x)\,dx \quad (11)$$

$$n(x) = \int_0^H x^2 \exp(-x)\,dx - 0{,}5\int_H^\infty x(x-H)\exp(-x)\,dx \quad (12)$$

Taking into account (4) and (5) we obtain

$$\int_0^\infty x^2 \exp(-x)\,dx = 2 \quad (13)$$

$$\int_H^\infty x(x-H)\exp(-x)\,dx = \int_H^\infty x^2\exp(-x)\,dx - H\int_H^\infty x\exp(-x)\,dx \quad (14)$$

$$= (2 + 2H + H^2)\exp(-H) - H(1+H)\exp(-H) = (2+H)\exp(-H)$$

By normalizing to one the base value (13), which corresponds to a homogeneous medium, we obtain

$$n(H) = 1 - 0{,}25\,(2+H)\exp(-H) \quad (15)$$

The change in the neutrons concentration near the boundary calculated in the first approximation is presented on Figure 6. At the distance to the boundary H/λ = 3,27, the neutron concentration and, accordingly, the neutron flux decreases by 5%.

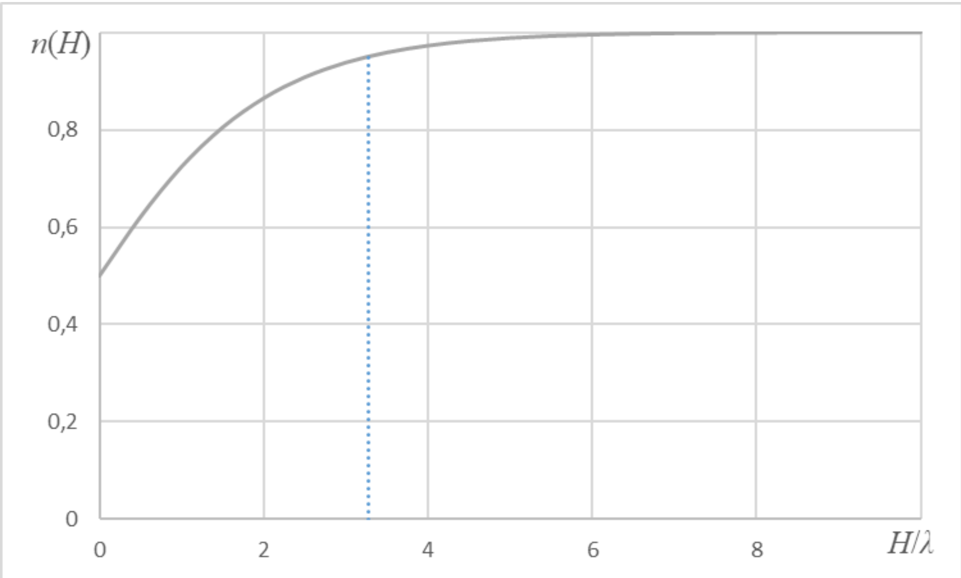

Figure 6: Change in neutron concentration $n(x)$ near the H/λ boundary.
The first approximation.

The first approximation was obtained based on the uniform distribution of the neutron concentration. The second approximation can be obtained based on the obtained first approximation. By continuing the iterative calculation of the following approximations, the distribution of neutron concentrations near the boundary with the absorbing shield can be updated.

The angle at which neutrons intersect the shield plate depends on the distance H and affects the effective shield thickness and BUF. The resulting distribution will also affect the efficiency of the shield. Therefore, the next steps in the development of the considered model will be an analytical assessment of the penetration and attenuation of the diffusion neutron flux in the shield.

4 CONCLUSION

The formation of neutron diffusion flux based on the scattering process in a homogeneous medium has been studied. The density of the diffusion flux of neutrons at a certain point is determined by how much previous scattering occurs around it. The effect of changes in the number of previous scatterings is noticeable at a distance of 6,30 λ and is most sensitive at a distance of 2λ, and apparently greater than at smaller distances.

The first approximation of the distribution of the neutron flux near the boundary with the absorbing shield is obtained. The exact form of that distribution can be obtained by subsequent iterations.

The formation of the neutron diffusion flux in front of the absorbing shield has a significant effect on the efficiency of the shield and, accordingly, on the BUF.

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