Extension of the WIMSD-5B Code for Multi-Zone Fuel Modelling Using Geometry-Consistent Dancoff Factors

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ABSTRACT

Deterministic reactor-physics codes remain important tools for routine reactor analysis, burnup calculations, and few-group constant generation. Within this class, the WIMS family of codes continues to be of practical interest, but its applicability must be extended when geometries fall outside its original standard scope. One such case is the annular, multi-zone fuel used in the RB and RA research reactors in Vinča. For this geometry, the standard resonance treatment in WIMSD-5B does not preserve the correct neutron escape-and-return behaviour of cylindrical shell-type fuel, which leads to an overestimation of the corresponding Dancoff factor and affects resonance self-shielding and the resulting group-averaged cross sections.

In this work, a procedure for extending the applicability of WIMSD-5B to annular multi-zone fuel analysis is developed using geometry-consistent Dancoff factors. The procedure is based on the VEGA2DAN sequence of the VEGA-2 lattice-physics code. VEGA2DAN determines effective Dancoff factors on the basis of the equivalence principle by preserving the absorption self-shielded cross section in a selected energy range between the real heterogeneous annular geometry and an equivalent geometry used in the resonance treatment. The resulting Dancoff factors were supplied to WIMSD-5B without modification of its core solution algorithms.

The methodology was demonstrated on the RA reactor fuel element, for which both a detailed three-dimensional model and an equivalent deterministic one-dimensional multizone model were constructed. Burnup calculations were benchmarked against VEGA-2/ORIGEN-2.2, TRITON from SCALE-5.1, MCNP-5/ORIGEN-2.2, and MCNPX-2.7/CINDER'90, the last of these being used as the principal reference methodology. Calculations were performed with ENDF/B-VI.6 and ENDF/B-VII.0 data.

The modified WIMSD-5B approach reproduced the burnup dependence of the infinite multiplication factor with acceptable deviations for the present heavy-water application, while very good agreement was obtained for the evolution of ^{235}U concentration. The ^{134}Cs/^{137}Cs concentration ratio proved to be a more sensitive indicator of resonance-treatment quality, confirming that the remaining limitations of the equivalence-based model are most visible in isotopic quantities influenced by resonance interference effects.

Keywords: *resonance absorption, Dancoff factor, cylindrical shell-type fuel, WIMSD-5B, MCNP-5*

1 INTRODUCTION

Well-established deterministic reactor-physics codes continue to play an important role in practical cell calculations, burnup analysis, and few-group constant generation. Although continuous-energy Monte Carlo methods provide the most reliable treatment of neutron transport for geometrically detailed systems, their computational cost generally limits their role to reference and validation calculations, while multigroup deterministic methods remain indispensable for

routine analysis and comparative studies. Within this class of codes, the WIMS modular reactor physics code [1] holds a particularly important place, as one of the earliest complete multigroup transport methodologies developed beyond older approaches that treated the spatial and energy dependence of the neutron field in an overly simplified way. An additional reason for its long-standing relevance is that WIMS has been one of the few reactor lattice codes available in the public domain on non-commercial terms, which made it accessible to a broad range of institutions, including those in countries without extensive commercial-code infrastructure. For those reasons, extending the applicability of validated WIMS-based methods to geometries and physical situations outside their original standard scope remains a relevant and worthwhile task.

A concrete example of such a need is provided by the RB [2] and RA [3] research reactors in Vinča, whose fuel elements have cylindrical shell-type (annular) geometry, characteristic for Soviet-produced fuel developed in the 1950s and 1960s, although the relevance of such geometries is not limited to these particular systems. Annular fuel concepts also appear in broader reactor-physics contexts, including studies of high-performance fuel aimed at increasing power density and improving safety margins relative to conventional PWR fuel [4], as well as degraded accident configurations in which damage to the fuel may lead to the release of fuel particles into the moderator, creating a situation in which resonant absorbers are present not only in the fuel, but also in the moderator [5]. At the same time, modern burnup calculations increasingly require multi-zone fuel representations in order to track the spatial redistribution of ^{235}U, the buildup of other fissile and absorbing nuclides [6], and the behaviour of burnable absorbers [7].

In deterministic cell calculations, the real three-dimensional fuel configuration cannot in general be treated directly, but must instead be reduced to an equivalent repeated lattice cell that can be analysed in one- or two-dimensional form. Although such an approach is generally adequate for neutron slowing-down and thermalization calculations, the resonance energy range requires additional treatment. In resonance self-shielding calculations, the usual first approximation is to consider a single fuel region embedded in an infinite moderator, while the effect of neighbouring fuel regions is accounted for by the Dancoff correction [8]. This issue is particularly important in the case when fuel element is not a simple solid rod but a more complex annular, multi-zone configuration, as is the case for the RB and RA fuel elements. Under such conditions, an accurate determination of the Dancoff factor becomes essential, because it directly affects the resonance treatment and the resulting group-averaged cross sections.

While annular configurations can be treated within the SCALE code system framework [9], they have historically represented a challenge for the WIMS codes. In particular, WIMS-D/4 [10], which was widely used in developing countries, was not able to treat tubular fuel adequately. The various improvements of WIMS-D/4 by different laboratories, such as WIMS-AML [11], and the introduction of WIMSD-5B [12] made direct treatment of tubular fuel possible. Even so, WIMSD-5B does not inherently retain a physically consistent resonance treatment for annular fuel shells, which leads to an overestimation of the Dancoff factor.

For these reasons, a dedicated procedure for determining Dancoff factors for annular fuel geometries was prepared and implemented through the VEGA2DAN program [13], which is part of the domestic VEGA-2 design-oriented lattice physics code [14]. In the present work, VEGA2DAN was used specifically to supply WIMSD-5B with the appropriate Dancoff factors for the individual fuel shells, thereby extending the applicability of WIMSD-5B to annular and multi-zone fuel configurations without modifying its core solution algorithms.

After establishing this procedure, the resulting WIMS-based approach was benchmarked against a broader computational toolset comprising VEGA-2 coupled with ORIGEN-2.2 [15], TRITON [16] from the SCALE-5.1 code package [17], MCNP-5 [18] with ORIGEN-2.2 integrated by the MOCUP driver [19], and MCNPX-2.7 [20] with CINDER'90 library [21, 22], the last of these being used as the principal reference methodology because of its comprehensive depletion treatment and large transmutation network. Burnup calculations were performed for the RA reactor fuel element with annular geometry, and the obtained results were compared with respect to both

the influence of the nuclear data library, ENDF/B-VI.6 [23] versus ENDF/B-VII.0 [24], and the influence of the computational methodology itself.

2 METHODOLOGY

2.1 Computational codes

The applied methodologies include multigroup deterministic methods, represented by WIMSD-5B and VEGA-2, as well as more advanced transport-depletion sequences, represented by TRITON from the SCALE-5.1 code system and the MCNP-based Monte Carlo approaches. The purpose of this comparison was not only to evaluate the performance of the developed WIMS-based methodology, but also to distinguish differences caused by the computational approach from those associated with the underlying nuclear data library. Among the considered approaches, the MCNPX-2.7/CINDER'90 sequence was treated as the main reference depletion methodology. Its role was to provide a high-fidelity transport-burnup solution for comparison with the more design-oriented multigroup methods. MCNP-based methods are capable of representing the full detailed geometry directly and therefore serve as reference tools for validation of reduced cell models, even though their computational cost is much higher than that of deterministic cell codes. Because WIMSD-5B and VEGA2 are central to the present work, they are described here in greater detail.

2.1.1 WIMSD-5B

WIMSD-5B was used as the central multigroup deterministic code in the present work. It solves the integral transport equation in the unit cell by the collision probability method with flat-flux approximation over the basic cell regions. The transport solution is carried out up to the coolant region, while the outer moderator is treated approximately through a diffusion-based balance relation coupled at the coolant boundary. This stage provides the cell flux and the cross-section data needed for subsequent processing.

In the resonance range, WIMSD-5B applies a treatment based on the equivalence principle, in which the heterogeneous problem is represented through an equivalent combination of homogeneous cases. In that procedure, the resonance integrals depend on geometrical characteristics of the fuel together with the associated Bell and Dancoff factors [25], which may either be calculated internally or supplied by the user.

The transport equation in full geometry, defined by the user, is solved using the resonance-processed cross sections obtained in the previous step. At this level, the number of energy groups can be reduced to a smaller set specified in the input, while the coefficients used in the transport solution are taken from the unit cell calculations, including the resonance-processed cross sections. WIMSD-5B allows several geometry models and numerical solution options here, including discrete-ordinates and collision-probability-based treatments. The output of this calculation is then used for the generation of few-group constants suitable for nodal or diffusion-style whole-core calculations.

For depletion calculations, WIMSD-5B uses a burnup model with 54 explicit fission products and one pseudo-fission product. To reduce the computational cost of burnup calculations, the code uses an internal loop in which the flux spectrum is corrected by solving the diffusion equation for a homogenized material mixture, instead of repeating the full lattice calculation at every burnup step. Since burnup affects mainly the absorption cross sections, while transport cross sections change more slowly, several shorter depletion steps can usually be performed before a new full lattice calculation is required.

2.1.2 VEGA-2

VEGA-2 is an assembly spectrum code developed for producing few-group averaged cross sections for a wide range of reactor types. Its theoretical basis is integral transport theory in the first-flight collision-probability formulation. The assembly calculation is preceded by a sequence of spectrum calculations performed in the basic library energy-group structure. A series of different spectrum geometries is used to represent the typical local situations within the assembly, and the resulting spectra are then used to form few-group macroscopic cross sections for the assembly calculation itself. A comprehensive set of collision-probability routines is available for different geometries, including one-dimensional slab, cylindrical, and spherical cases, as well as two-dimensional rod-array and cluster configurations.

In the resonance range, VEGA-2 applies different treatments in the unresolved and resolved resonance regions. In the unresolved resonance range, the code uses the subgroup method. In the resolved resonance range, several levels of treatment are available, depending on the required balance between accuracy and computational cost: the subgroup method, the generalized Jeanpierre–Livolant formalism with resonance-interference corrections (GIJ-RIF) [26,27], exact treatment in the fuel and cladding regions with a 1/E approximation in the moderator, and exact treatment throughout the geometry. The latter three options give cross-sections that differ by about 1% relative to the subgroup-based treatment, corresponding to roughly 100 pcm difference in the infinite multiplication factor.

The full assembly calculation in VEGA-2 is then performed in several broad energy groups using the cross sections obtained from the preceding spectrum calculations. The assembly is treated without prior pin-cell homogenization, and the spectra calculated for the different cell and subregion types are used to generate the few-group macroscopic constants required for the final assembly solution. In addition, VEGA-2 allows further subdivision of regions when needed, which makes it more flexible in representing the local flux distribution around annular fuel and moderator regions. The principal output includes reactivity, fluxes, reaction rates, and few-group cross sections for further core calculations.

For depletion calculations, the VEGA-2 model contains 95 explicit fission products and two pseudo-fission products, but it can also be coupled with programs such as ORIGEN-2.2. In that case, VEGA-2 provides cross sections for the most important nuclides, while ORIGEN performs the isotopic evolution; the remaining nuclides are taken directly from a preexisting ORIGEN library.

2.2 Dancoff factor calculation

A key difficulty in applying WIMSD-5B to the present problem arises in the resonance treatment of annular fuel. Although annular geometry can be entered in the code input, the standard WIMSD-5B resonance treatment does not inherently preserve the correct neutron escape-and-return behaviour of annular fuel shells. In the resonance calculation, geometric effects are represented through equivalence-theory parameters, including the Dancoff and Bell factors, within an escape-probability framework. For annular multi-zone fuel, this parameterization does not fully represent the shell-wise absorber-moderator arrangement, which can lead to an overestimation of the corresponding Dancoff factor. As a consequence, inconsistencies are introduced into the resonance self-shielding treatment and the resulting group-averaged cross sections. Since resonance-group cross sections depend directly on this treatment, a physically consistent correction is required before the code can be applied reliably to annular multi-zone fuel. Because the Dancoff factor represents the probability that a neutron leaving one fuel region reaches another absorber region without collision in the intervening moderator, its value depends strongly on the actual fuel geometry and lattice arrangement; for annular multi-zone fuel, this requires shell-wise treatment rather than the use of a single approximate value.

In the present work, this correction was implemented through the VEGA2DAN sequence of the VEGA-2 lattice-physics code. VEGA2DAN is a procedure for effective Dancoff factor determination based on the equivalence principle. It equalizes the absorption self-shielded cross section in the chosen energy range of a heterogeneous medium to an equivalent medium containing the single fuel region, in which the presence of other fuel regions is taken into account by applying the effective Dancoff factor. In this way, the procedure not only preserves the collision-probability aspect of the problem, but it accounts for both resonance interference and overlap effects. The superiority of this procedure is apparent in complicated, irregular geometries where the other methods are not applicable, and where the flux solution by collision probability method is possible.

In practical terms, the Dancoff factor as a function of lattice pitch was first determined by VEGA2DAN for both the annular reference geometry and an equivalent plate geometry. For the actual pitch of the annular system, the corresponding Dancoff factor was obtained, and the equivalent slab pitch was then selected so that the plate representation reproduced the same Dancoff factor. It should be emphasized that the annular fuel-shell geometry itself was retained in the reduced cell model, while the WIMSD-5B resonance treatment was supplied with the shell-wise Dancoff factors and the corresponding equivalent plate-geometry parameters obtained from the VEGA2DAN procedure. In this way, the annular fuel was not replaced globally by a plate model; rather, the equivalent plate representation was introduced only at the level of the resonance treatment, thereby avoiding direct reliance on the standard WIMSD-5B slab-based approximation. The physical basis of this procedure lies in the equivalence-theory treatment used by WIMSD-5B in the resonance range, where the influence of geometry is represented through an effective escape-related parameter expressed in terms of fuel geometry together with Bell and Dancoff corrections. If the fuel geometry is represented inconsistently, this parameter is also inconsistent, and the resulting self-shielded resonance cross sections are distorted. In VEGA2DAN, neutron absorption in both the real and equivalent media is evaluated by a fine-group elastic slowing-down calculation, so that the resulting effective Dancoff factor preserves the resonance-range neutron balance governing the self-shielding calculation.

Using this approach, VEGA2DAN was applied to determine the appropriate shell-wise Dancoff factors for the annular RA fuel configuration and to supply those values to WIMSD-5B, thereby linking the real annular multi-zone geometry to the resonance treatment available in WIMSD-5B without modification of the code's core solution algorithms.

2.3 Calculation model of the RA fuel element

The benchmark problem considered in the present study was an RA reactor fuel element with annular, multi-zone fuel geometry. A three-dimensional model of the RA fuel element is shown in Figure 1, its radial dimensions are given in Table 1, while the material composition of the fresh fuel element is given in Table 2, Table 3 and Table 4 [28]. In this model, the detailed design features of the RA fuel element were represented as closely as possible in order to provide a physically realistic basis for the benchmark calculations.

The complexity of fuel depletion and decay analysis for the RA reactor arises primarily from the effect of the end region between adjacent fuel elements in the core. In general, this problem can be treated in two ways. The first is to use Monte Carlo methods, which are able to model the full three-dimensional geometry of an asymmetric fuel element of finite height, such as the one shown in Figure 1. The second is to construct an equivalent one- or two-dimensional model in which a representative unit cell is able to be defined and calculated using the deterministic lattice-physics codes.

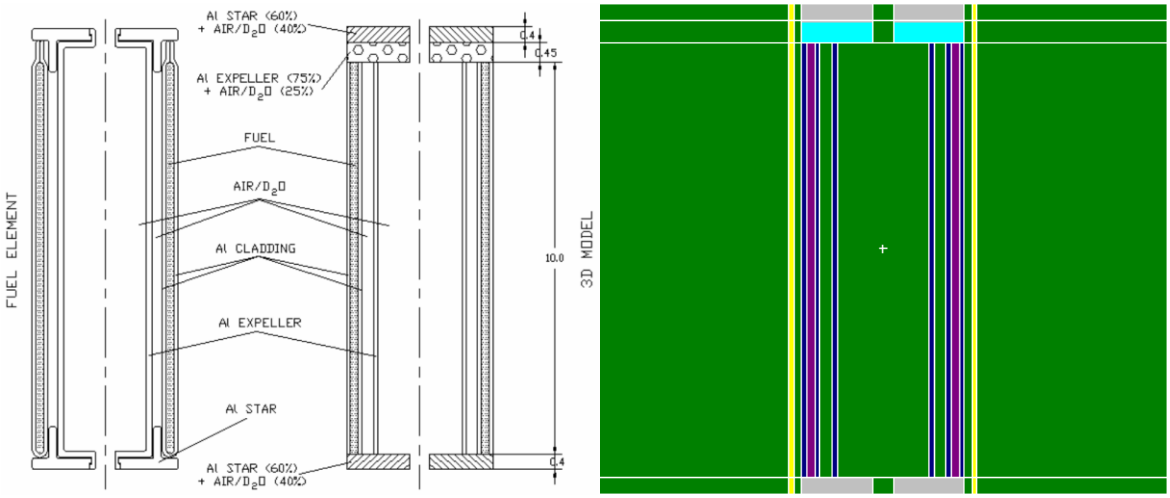

Figure 1: Vertical cross-sections of the RA reactor fuel element: original design drawing and corresponding 3D geometrical model used in the present study. The fuel is clad on both the inner and outer surfaces by aluminium, while the central aluminium tube (expeller) regulates the coolant flow. The upper and lower end regions contain aluminium structural parts ("stars") connected to the expeller.

Table 1: Radial dimensions of the RA fuel element in the detailed 3D model and the equivalent 1D deterministic model

Zone	Material	Radius [cm]	
		Detailed 3D model	Equivalent 1D model
1	Heavy Water	1.05000	1.00677
2	Clad (Al-SAV)	1.17500	1.17500
3	Heavy Water	1.45000	1.42021
4	Clad (Al-SAV)	1.55000	1.55000
5	Fuel (2 % or 80 % ^{235}U)	1.75000	1.75000
6	Clad (Al-SAV)	1.86000	1.89093
7	Heavy Water	2.05000	2.05000
8	Fuel Channel (Al-SAV)	2.15000	2.16266
9	Heavy Water (Outer Moderator)	Square Cell (Pitch = 13 cm)	7.79664

Table 2: Atomic number densities in the RA fuel element (2% ^{235}U, density 17.42 g/cm^3)

Nuclide	Atomic number density [10^{24} cm^{-3}]
^{234}U	$5.37300 \cdot 10^{-6}$
^{235}U	$8.96136 \cdot 10^{-4}$
^{238}U	$4.31300 \cdot 10^{-2}$
^{10}B	$4.82854 \cdot 10^{-8}$
^{11}B	$1.94355 \cdot 10^{-7}$
C	$6.12127 \cdot 10^{-4}$
N	$4.09769 \cdot 10^{-5}$
Al	$1.16666 \cdot 10^{-5}$
Si	$4.07223 \cdot 10^{-5}$
Mn	$2.57839 \cdot 10^{-6}$
Fe	$3.23923 \cdot 10^{-5}$
Ni	$5.18438 \cdot 10^{-6}$
Cu	$2.75969 \cdot 10^{-6}$

Table 3: Atomic number densities in the aluminium Al-SAV (density 2.729 g/cm^3)

Nuclide	Atomic number density [10^{24} cm^{-3}]
^{10}B	$2.11756 \cdot 10^{-8}$
^{11}B	$8.52347 \cdot 10^{-8}$
Mg	$3.78655 \cdot 10^{-4}$
Al	$5.99200 \cdot 10^{-2}$
Si	$4.97380 \cdot 10^{-4}$
Ti	$2.74592 \cdot 10^{-6}$
Mn	$6.58114 \cdot 10^{-7}$
Fe	$5.88548 \cdot 10^{-5}$
Ni	$4.20028 \cdot 10^{-7}$
Cu	$1.18966 \cdot 10^{-6}$
Cd	$2.92399 \cdot 10^{-9}$

Table 4: Atomic number densities in the heavy water (0.5% H_2O)

Nuclide	Atomic number density [10^{24} cm^{-3}]
^{1}H	$3.2946 \cdot 10^{-4}$
^{2}H	$6.5563 \cdot 10^{-2}$
^{16}O	$3.2946 \cdot 10^{-2}$

For deterministic calculations, the end-region problem was treated by introducing a multi-region cylindrical Wigner-Seitz cell with white boundary conditions at the cell boundary, that is, isotropic reflection at the outer cylindrical surface. This representation corresponds to an infinite array of infinitely long cells in the axial direction. Owing to radial symmetry in the plane perpendicular to the cylinder axis, the model can be treated in one-dimensional cylindrical geometry. In this equivalent model, the aluminium from the end region, including the expeller and star-shaped structural parts, was redistributed into the inner and outer aluminium cladding, while the outer cylindrical moderator region was enlarged by the volume of heavy water originally present in those regions, so that the total volume of both aluminium and moderator was preserved. The resulting deterministic representation was therefore not intended to reproduce every geometric detail exactly, but to preserve the most important neutronic characteristics of the real fuel element, especially those relevant for resonance absorption and few-group constant generation. The final result was a nine-zone unit-cell model shown in Figure 2, with dimensions also given in Table 1.

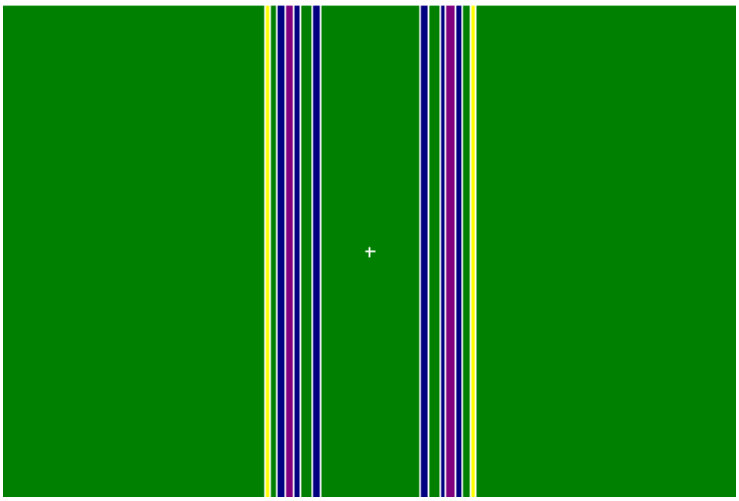

Figure 2: Vertical cross-section of the equivalent 1D multizone deterministic model of the RA fuel element used for deterministic calculations. The end-region aluminium and moderator volumes are redistributed so that the most important neutronic characteristics of the original fuel element are preserved.

In order to make the benchmark as consistent as possible between deterministic and Monte Carlo methods, a dedicated treatment of the end region was adopted for the Monte Carlo calculations as well. Instead of performing calculations on a detailed three-dimensional model of fixed height, a unit cell was constructed directly from the detailed three-dimensional model by centering the cell at the boundary between two adjacent fuel elements. In this way, a repeatable sequence was obtained that could be extended infinitely in the axial direction under the reflecting boundary conditions, that is by preserving the general reflection rule at the outer cylindrical surface, while still preserving radial symmetry in the plane perpendicular to the cylinder axis. The resulting unit-cell model is shown in Figure 3: Unit-cell model used in the MCNP and KENO-V.a calculations, constructed directly from the detailed 3D geometry by centering the cell at the boundary between two adjacent RA fuel elements. This representation defines a repeatable sequence that can be extended in the axial direction for unit-cell calculations. and in the present work it was used as the geometrical input for the MCNP and KENO-V.a calculations.

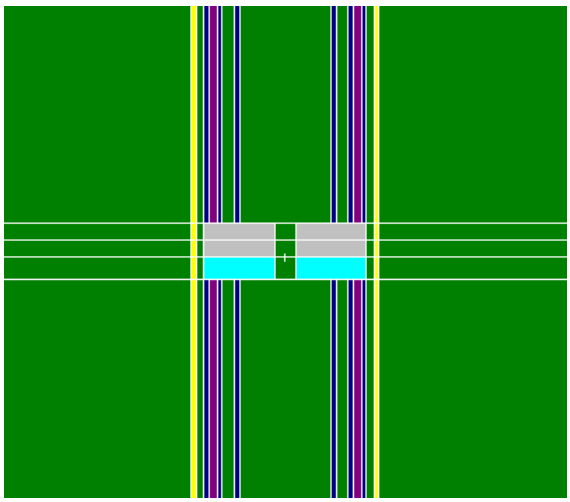

Figure 3: Unit-cell model used in the MCNP and KENO-V.a calculations, constructed directly from the detailed 3D geometry by centering the cell at the boundary between two adjacent RA fuel elements. This representation defines a repeatable sequence that can be extended in the axial direction for unit-cell calculations.

2.4 Computational setup

The benchmark calculations were performed under a common burnup scheme, while the transport, depletion, and library settings were selected according to the characteristics of each code system.

For WIMSD-5B, a 69-group library was used. The transport equation in the full geometry was solved by the discrete ordinates method (S16), while burnup was calculated using the built-in depletion model containing 54 explicit fission products and one pseudo-fission product.

VEGA-2 employed its operating 55-group library. The slowing-down equation in the unresolved resonance range was solved with exact treatment in the fuel and cladding regions, while the moderator was represented by a 1/E approximation. For burnup calculations, VEGA-2 was coupled with ORIGEN-2.2: VEGA-2 supplied cross sections for the most important nuclides (37 actinides and 85 fission products), while the remaining nuclides were taken from a domestically prepared ORIGEN-2.2 library originally developed for the RA research reactor.

In the TRITON sequence, resonance self-shielding was first obtained in a one-dimensional cylindrical cell model by solving the slowing-down equation in the resolved resonance range with ultrafine groups, effectively in a continuous-energy representation, using the discrete ordinates method (S8). The subsequent neutron transport simulation was performed with KENO-V.a with a 238-group library and reflecting boundary conditions [29]. The isotopic evolution during burnup was then calculated with ORIGEN-S [30]. As in the VEGA2-based methodology, the most important nuclides (37 actinides and 85 fission products) were treated explicitly, while the remaining nuclides were calculated on the basis of the three-group spectrum distribution in the fuel and the corresponding three-group library included in the SCALE package. The SCALE 5.1 version used in this work does not include support for ENDF/B-VII libraries, so calculations were only done for the ENDF/B-VI.6 library.

MCNP-5 and MCNPX-2.7 both used Monte Carlo calculations with a continuous-energy representation of cross sections, while differing in the depletion methodology. In the MCNP5 sequence, burnup was calculated by coupling the Monte Carlo calculation to ORIGEN-2.2 through the MOCUP driver, in essentially the same way as in the VEGA-2 based methodology. In MCNPX-2.7, depletion was performed with the CINDER'90 library, in which the transmutations of approximately 3400 nuclides are followed. Owing to this more comprehensive treatment of nuclide evolution, the MCNPX-2.7/CINDER'90 sequence was taken as the principal reference methodology in the present work.

The same burnup-step structure was used in all calculations. The adopted sequence consisted of an initial step of 1 day, followed by four steps of 2 days, one step of 4.3 days, and a final step of 13.4 days. This choice was made so that each step corresponded approximately to a burnup increment of 250 MWd/t, thereby preserving a consistent comparison basis among the considered methodologies. Regarding burnup integration, WIMSD-5B, and MCNPX-2.7/CINDER'90 employed the predictor-corrector scheme, whereas VEGA2, TRITON and MCNP-5/ORIGEN-2.2 used a predictor-only scheme.

3 RESULTS

The same RA fuel-cell benchmark problem was analysed by all considered methodologies in order to compare the predicted multiplication factor, isotopic evolution, and other burnup-dependent quantities on a common basis. The comparison was intended to distinguish, as clearly as possible, the effects of the computational methodology from those associated with the evaluated nuclear data, while assessing the performance of the developed WIMSD-5B based approach against both deterministic and Monte Carlo reference methods. As stated earlier, the MCNPX-2.7/CINDER'90 sequence was used as the principal reference methodology.

The most relevant results are presented through the evolution of the multiplication factor during burnup, the depletion of ^{235}U, and the $^{134}Cs/^{137}Cs$ concentration ratio.

3.1 Evolution of the infinite multiplication factor with burnup

The evolution of the infinite multiplication factor with burnup is shown in Figure 4 and Figure 5, where the results obtained with the considered methodologies are presented separately for the ENDF/B-VI.6 and ENDF/B-VII.0 nuclear data libraries. In addition, Table 5 summarizes the differences between the two libraries within the same code, while Table 6 gives the deviations of the individual methodologies relative to the MCNPX-2.7/CINDER90 reference sequence for the ENDF/B-VI.6 case. The tabulated comparisons were evaluated for two representative points: fresh fuel and average burnup, the latter corresponding to 6.9 GWd/t. The comparison is presented both graphically and in tabulated form in order to distinguish the influence of the computational methodology from that of the evaluated nuclear data.

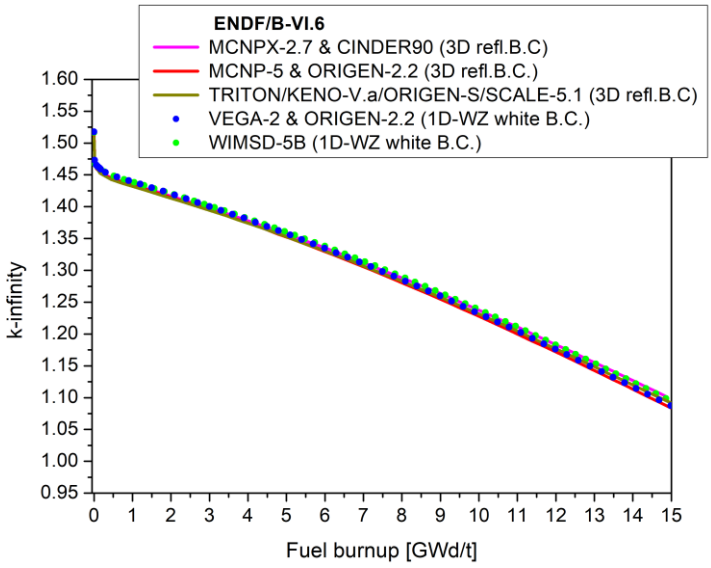

Figure 4: Burnup dependence of the infinite multiplication factor k_∞ obtained with the considered methodologies using the ENDF/B-VI.6 nuclear data library

Table 5: Differences between results obtained with the ENDF/B-VII.0 and ENDF/B-VI.6 cross section data using various fuel burnup analysis codes

Code	Δk_∞ (ENDF/B-VII.0 - ENDF/B-VI.6) [pcm]	
	Fresh fuel (0 GWd/t)	Average fuel burnup (6.9 GWd/t)
MCNPX-2.7/CINDER90	-533	-375
MCNP-5/ORIGEN-2.2	-544	-469
VEGA-2/ORIGEN-2.2	-523	-475
WIMSD-5B	-459	-409

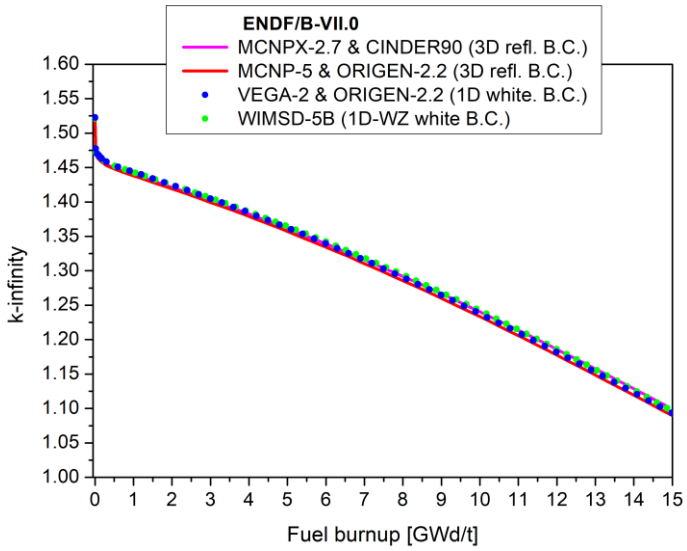

Figure 5: Burnup dependence of the infinite multiplication factor k_∞ obtained with the considered methodologies using the ENDF/B-VII.0 nuclear data library

Table 6: Differences between results obtained with various fuel burnup analysis codes for the ENDF/B-VI.6 cross section data

Code	Δk_∞ relative to MCNPX 2.7-CINDER90 [pcm]	
	Fresh fuel (0 GWd/t)	Average fuel burnup (6.9 GWd/t)
MCNP 5-ORIGEN-2.2	4	-642
TRITON (KENO-V.a/ORIGEN-S) SCALE-5.1	-307	-483
VEGA2-ORIGEN-2.2	550	-132
WIMSD-5B	475	243

A common trend observed in all methodologies is a monotonic decrease of the infinite multiplication factor with burnup, with a pronounced initial contribution from xenon buildup. The general agreement of the curves confirms that all considered methods reproduce the same overall depletion behaviour, while the remaining differences reflect the combined effects of transport treatment, resonance processing, depletion methodology, and nuclear data selection.

The comparison between the ENDF/B-VI.6 and ENDF/B-VII.0 results shows that, at the beginning of burnup, the difference between the two libraries within the same code is of the order of -500 pcm. At average burnup this difference becomes more uniform among the methods and decreases to about -350 to -450 pcm, as shown in Table 5. This behaviour indicates that the library effect remains systematic throughout burnup, but is somewhat less pronounced after the initial phase.

When the same library is used and the methods are compared directly, most deviations remain within approximately ± 500 pcm in the initial part of burnup. For the present heavy-water reactor application, this level of agreement is regarded as acceptable. This interpretation is consistent with published heavy-water-related benchmark experience, where discrepancies of the order of several hundred pcm, often around 500 pcm, are reported for reactor-physics calculations and experiment-to-calculation comparisons [31].

The deviations between methodologies become more visible at average burnup, as seen in Table 6. This is consistent with the fact that the compared methods differ not only in transport and resonance treatment, but also in burnup integration procedure, depletion-library structure, and the number of explicitly tracked nuclides. In particular, differences between predictor-only and predictor-corrector depletion schemes, as well as differences in the treatment of minor nuclides and fission products, are expected to contribute increasingly as burnup progresses. The larger initial deviation of VEGA-2 and WIMSD-5B relative to the MCNPX-2.7/CINDER90 sequence is reduced significantly at average burnup, while the TRITON result remains consistently below the reference, indicating that methodological differences persist even when the same nuclear data library is used.

Overall, the results show that the modified WIMSD-5B methodology follows the reference trend well over the whole burnup range. The remaining differences are of the same order as those observed among the other deterministic approaches, which supports the conclusion that the geometry-consistent Dancoff-factor treatment improves the applicability of WIMSD-5B to annular multi-zone fuel without introducing deviations beyond the level expected for the present type of benchmark.

3.2 Burnup dependence of ^{235}U concentration

The evolution of the ^{235}U concentration with burnup is shown in Figure 6, where the results obtained with the considered methodologies are presented for the ENDF/B-VII.0 nuclear data library. This variable was selected because it provides a direct measure of fissile-material depletion and therefore allows comparison of the burnup treatment independently of the reactivity response.

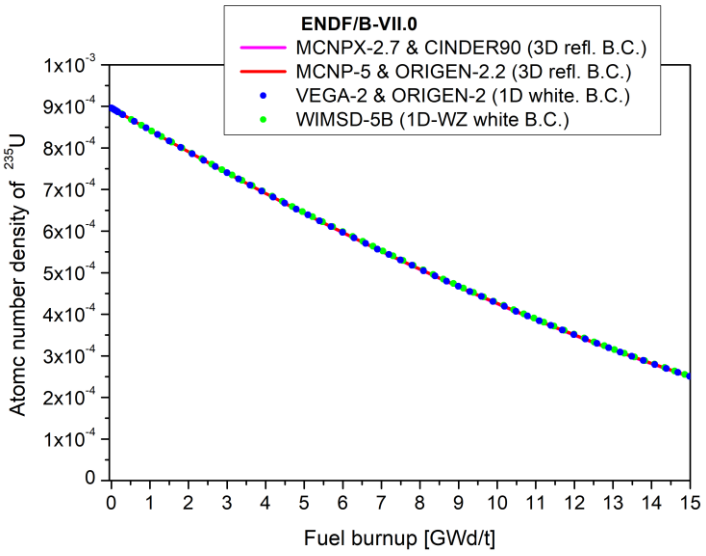

Figure 6: Burnup dependence of the ^{235}U atomic number density obtained with the considered methodologies using the ENDF/B-VII.0 nuclear data library

The compared methodologies show very good agreement in the predicted depletion trend over the whole burnup range. The differences between the curves remain small, indicating that the principal consumption of fissile material is described consistently across the applied methods,

despite the differences in transport methodology, resonance treatment, and depletion modelling. This indicates that the major part of the remaining discrepancies in reactivity cannot be attributed simply to differences in ^{235}U depletion, but must instead arise from the treatment of resonance absorption, fission-product buildup, and spectrum evolution during burnup.

The good agreement in ^{235}U concentration is particularly important in the present context because it confirms that the modified WIMSD-5B methodology does not introduce significant distortions in the basic fuel-depletion behaviour, so the underlying depletion of the principal fissile nuclide is reproduced in a physically consistent manner by all considered approaches.

3.3 ^{134}Cs/^{137}Cs concentration ratio

The evolution of ^{134}Cs/^{137}Cs concentration ratio with burnup is shown in Figure 7, where the results obtained with the considered methodologies are presented separately for the ENDF/B-VII.0 nuclear data library. This ratio was selected as a particularly sensitive indicator of the quality of resonance absorption treatment.

Unlike the infinite multiplication factor or the ^{235}U concentration alone, the ^{134}Cs/^{137}Cs ratio depends strongly on the local spectral behaviour in the epithermal region. The production of ^{134}Cs is governed by neutron capture in the fission product ^{133}Cs, whose absorption cross section contains a pronounced resonance at about 5.9 eV. At the same time, ^{238}U has a strong nearby resonance at about 6.7 eV. Because these resonances are close in energy, the local neutron flux in this region is strongly affected by resonance interference and by the associated spectral depression. As a result, the buildup of ^{134}Cs becomes highly sensitive to the accuracy with which the resonance treatment represents these effects.

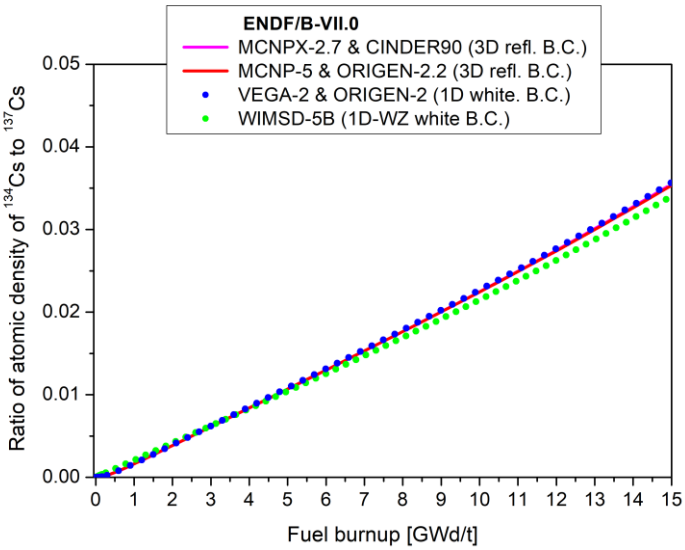

Figure 7: Burnup dependence of the ^{134}Cs/^{137}Cs concentration ratio obtained with the considered methodologies using the ENDF/B-VII.0 nuclear data library

The results show that the deviation of WIMSD-5B in this quantity is more pronounced than in the corresponding comparisons of the infinite multiplication factor or ^{235}U depletion. In particular, WIMSD-5B predicts a systematically lower ^{134}Cs/^{137}Cs ratio than the other considered methods. This behaviour can be interpreted physically as a limitation of the equivalence-based resonance treatment, which does not represent resonance interference effects with sufficient detail. In that sense, the ^{134}Cs/^{137}Cs ratio provides a more demanding test of the resonance treatment than integral quantities dominated by overall neutron balance or principal fissile depletion alone.

The observed discrepancy therefore supports the broader conclusion that improved geometric and resonance consistency is important not only for reactivity prediction, but also for isotopic prediction, especially for nuclides whose production depends on capture reactions in strongly structured resonance regions. Although the $^{134}Cs/^{137}Cs$ ratio is not the central metric of the present paper, it is useful because it shows that the consequences of resonance-treatment approximations are not restricted to infinite-multiplication-factor prediction alone. The same modelling assumptions also affect nuclide inventories relevant to burnup assessment and, in broader applications, to source-term estimation.

4 CONCLUSION

In this work, a procedure for extending the applicability of WIMSD-5B to annular, multi-zone fuel configurations was developed and tested on the RA reactor fuel element. The central problem addressed was the inconsistency of the standard WIMSD-5B resonance treatment when applied to annular fuel shells, where the built-in approximation does not preserve the correct neutron escape-and-return behaviour and therefore leads to an overestimation of the corresponding Dancoff factor. To overcome this limitation, shell-wise Dancoff factors were determined by the VEGA2DAN procedure and supplied to WIMSD-5B, thereby introducing a geometry-consistent resonance treatment without modification of the core solution algorithms of the code.

The modified WIMSD-5B approach was benchmarked against a broader set of deterministic and Monte Carlo burnup methodologies, including VEGA-2/ORIGEN-2.2, TRITON from SCALE-5.1, MCNP-5/ORIGEN-2.2, and MCNPX-2.7/CINDER'90, the last of these being used as the principal reference methodology. The comparison showed that the modified WIMSD-5B methodology reproduces the general burnup dependence of the infinite multiplication factor with deviations that remain within the range expected for the present heavy-water reactor application. In addition, the predicted evolution of the ^{235}U concentration was in very good agreement with the other methods, indicating that the proposed Dancoff-factor treatment does not introduce distortions in the basic fissile-material depletion behaviour.

At the same time, the comparison of the $^{134}Cs/^{137}Cs$ concentration ratio showed that differences in resonance treatment remain visible most clearly in isotopic quantities sensitive to resonance interference effects. The systematically lower values predicted by WIMSD-5B for this ratio indicate that, although the proposed modification substantially improves the applicability of the code to annular fuel, the limitations of the equivalence-based resonance treatment are still reflected in nuclides whose production depends on capture reactions in strongly structured resonance regions.

Overall, the results demonstrate that the proposed geometry-consistent Dancoff-factor treatment provides a practical and physically justified way to extend WIMSD-5B to annular multi-zone fuel analysis. In this form, the code can be used more reliably for burnup calculations of reactor fuel elements of cylindrical shell type, while preserving the computational simplicity and practical value that make WIMS-based methods relevant for routine reactor-physics studies.

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